

Vectors and Coordinates #1

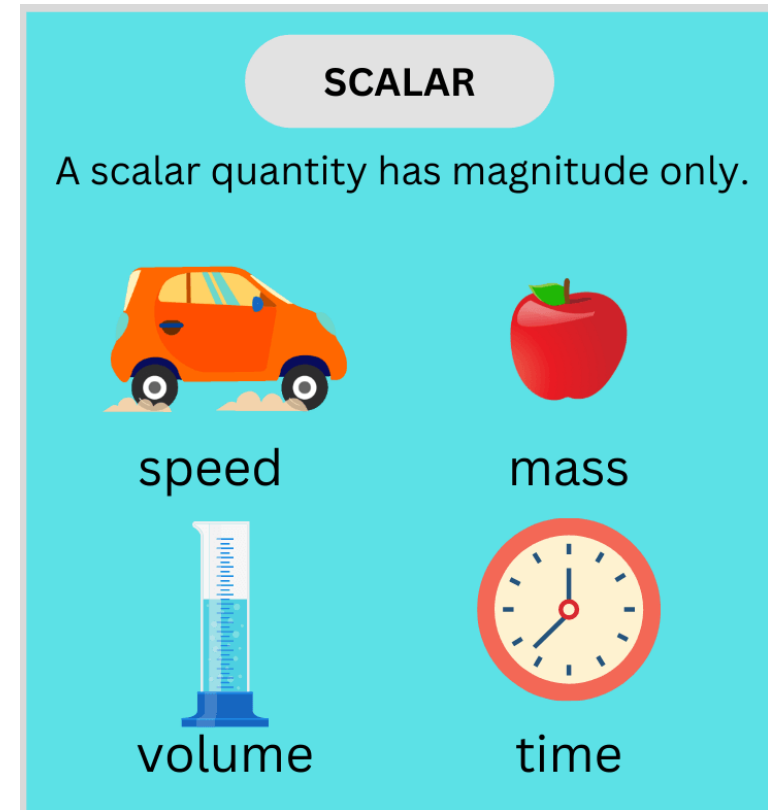


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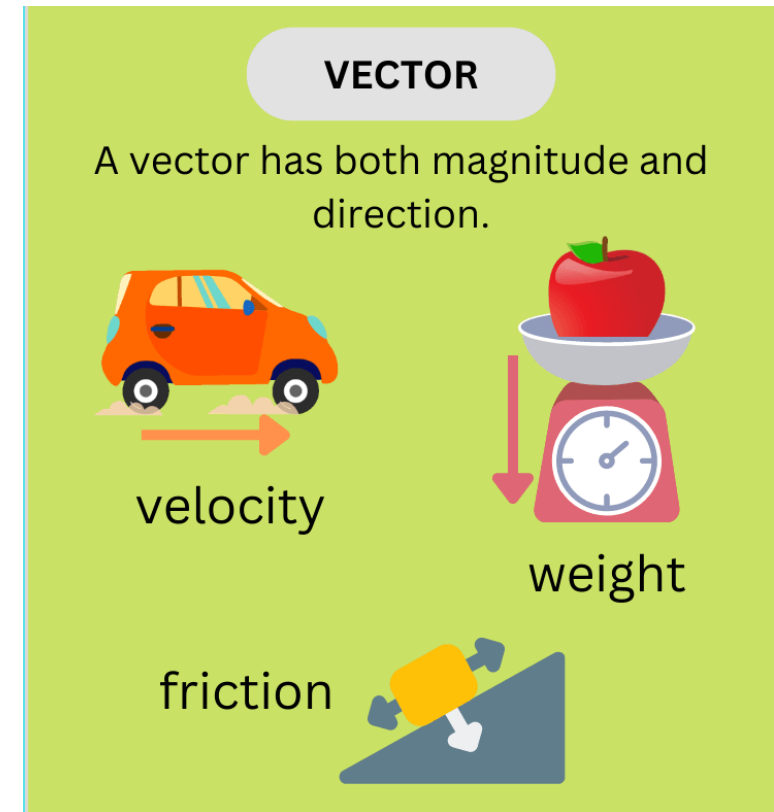
Scalar quantities

- **Definition:** Scalars are quantities that have magnitude only (size or amount), with no direction.
- **Examples:**
 - Mass (e.g. 5 kg)
 - Temperature (e.g. 22 °C)
 - Time (e.g. 3 s)
 - Speed (e.g. 10 m/s)
 - Energy (e.g. 200 J)
- **Key Point:** Scalars can be added or subtracted using normal arithmetic.



Vector quantities

- **Definition:** Vectors are quantities that have magnitude and direction.
- **Examples:**
 - Displacement (e.g. 5 m East)
 - Velocity (e.g. 20 m/s North)
 - Force (e.g. 10 N downward)
 - Acceleration (e.g. 3 m/s^2 to the left)
- **Key Point:** Vectors must be added using vector rules
- **Notation:** Often shown with arrows ($\rightarrow$) or bold letters



Scalar and vector quantities

Scalar Quantities (magnitude only)

Mass (kg)

Temperature ($^{\circ}\text{C}$, K)

Time (s)

Speed (m/s)

Energy (J)

Distance (m)

Volume (m^3)

Density (kg/m^3)

Power (W)

Pressure (Pa)

Vector Quantities (magnitude + direction)

Displacement (m, with direction)

Velocity (m/s, with direction)

Force (N, with direction)

Acceleration (m/s^2 , with direction)

Momentum ($\text{kg}\cdot\text{m}/\text{s}$, with direction)

Weight (N, acts toward Earth's centre)

Torque ($\text{N}\cdot\text{m}$, with rotation direction)

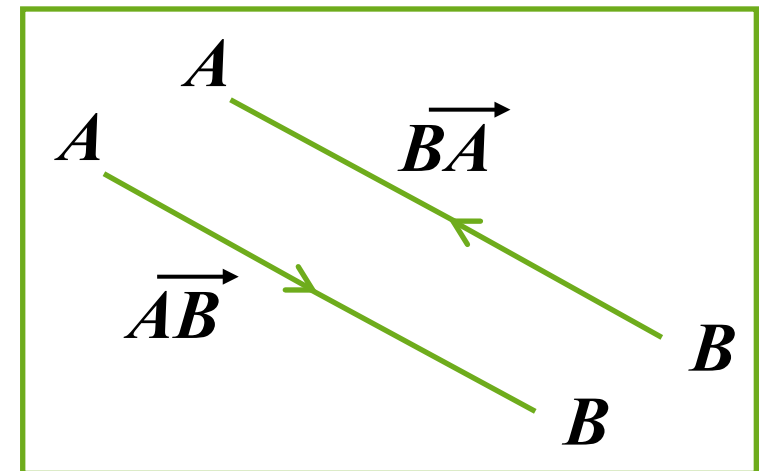
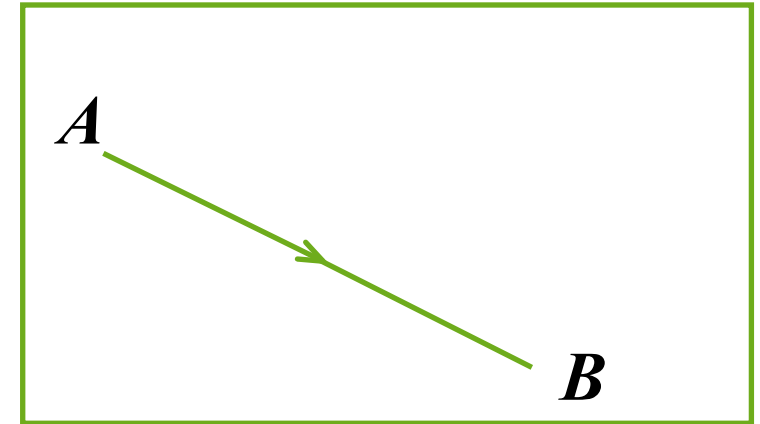
Electric Field (N/C, with direction)

Magnetic Field (Tesla, with direction)

Angular Velocity (rad/s, with rotation axis)

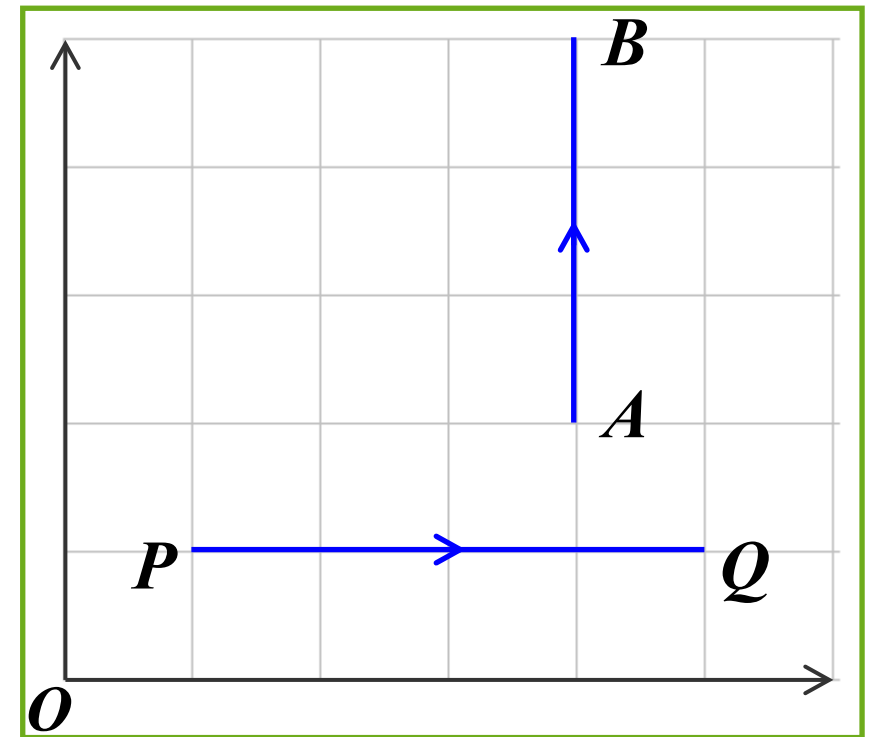
Drawing Vectors and Notation

- Vectors can be drawn as a segment between two points with an arrow
- They can then be written as $\overrightarrow{AB}$
- Arrows that run in opposite directions can be written as inverses:
- $\overrightarrow{AB} = \overrightarrow{BA}$



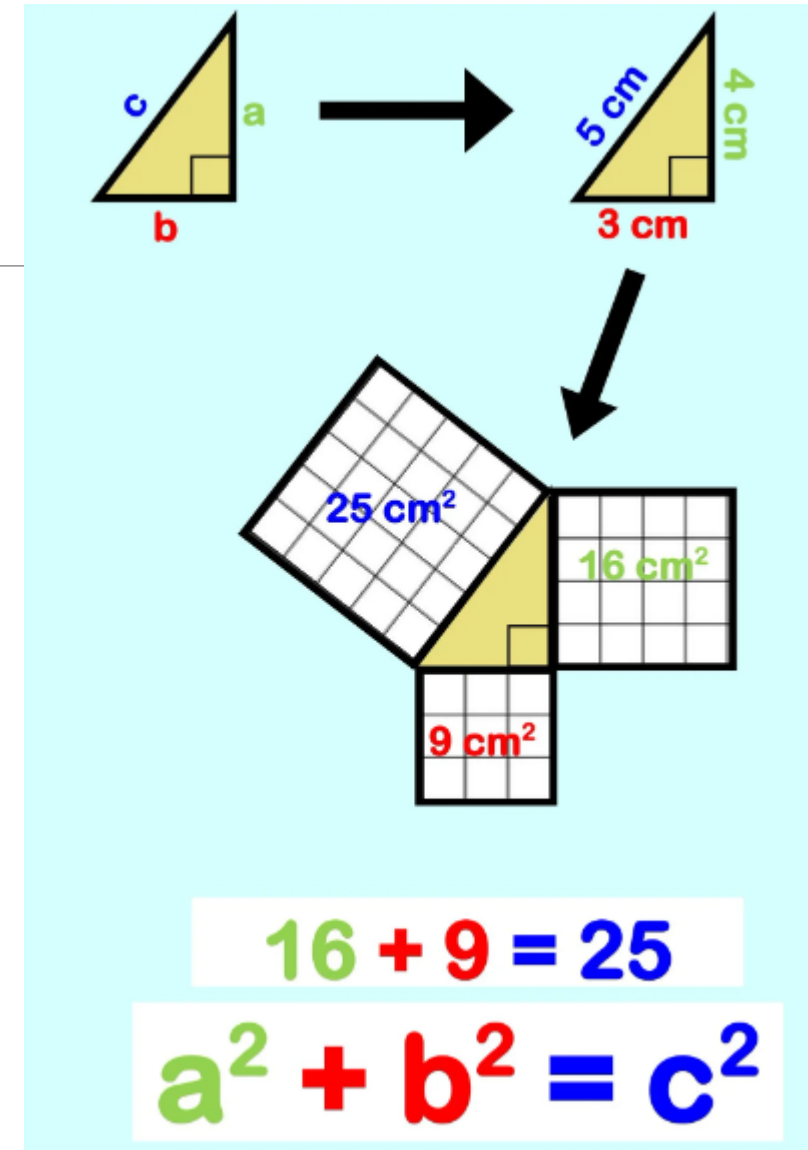
Magnitude of a vector

- The magnitude of a vector is based on the length of the line
- The grid on this slide has 1cm squares
- $\overrightarrow{AB}$ has the magnitude 3 so we can write $AB = 3$



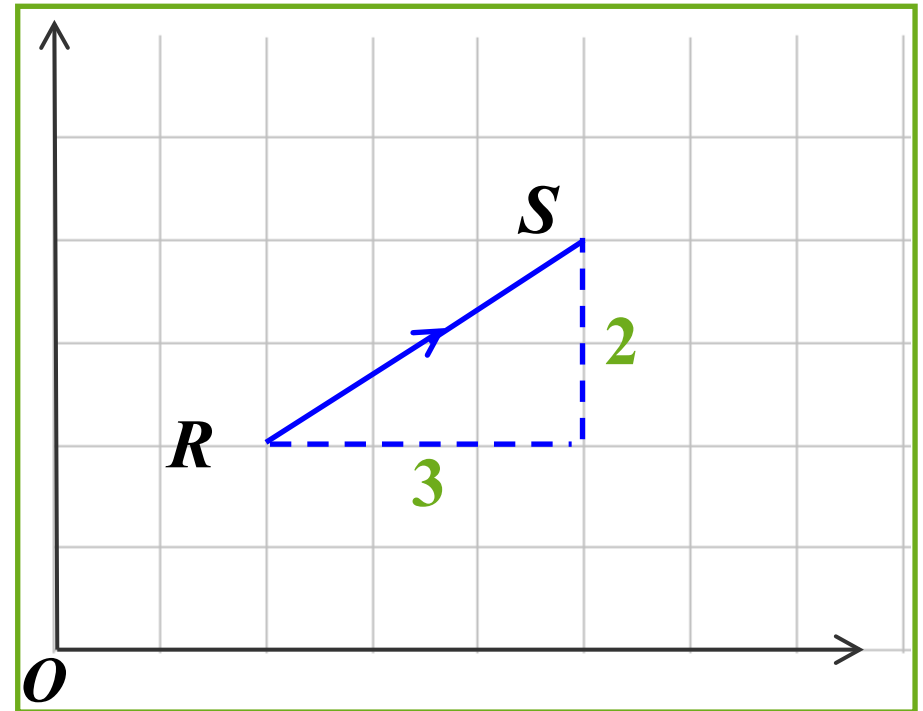
Pythagorean Theorem Recap

- We know $a^2 + b^2 = c^2$
- This can be applied to vectors too
- Vectors are essentially just hypotenuses of triangles
- So to work out the magnitude of a vector we can use Pythagorean theorem

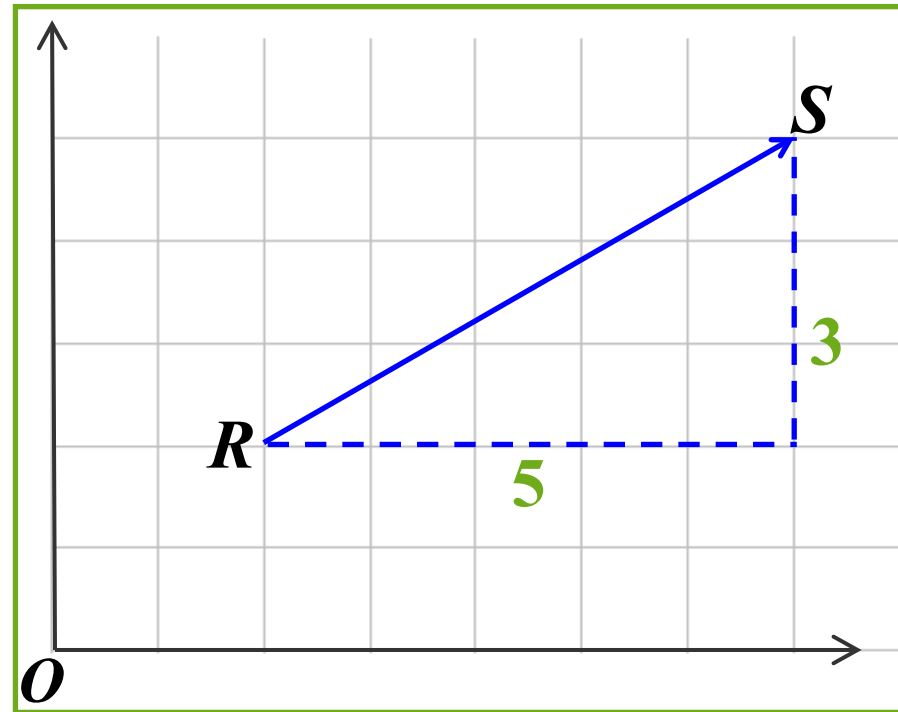


Calculating the magnitude of vectors

- Using Pythagoras' theorem we can take x and y and work out the magnitude
- $RS^2 = 3^2 + 2^2$
- $RS^2 = 13$
- $RS \approx 3.61$



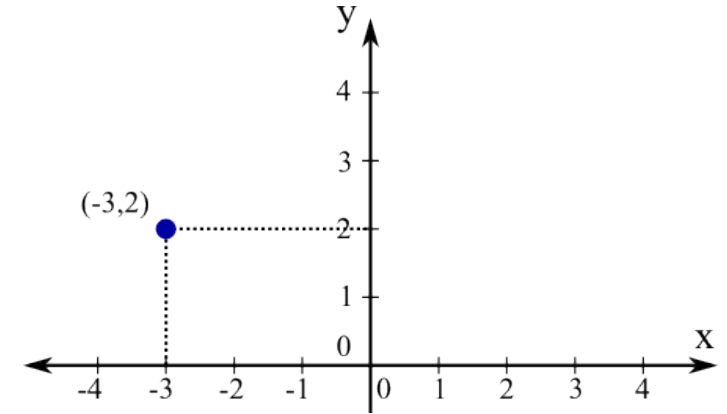
Calculate the magnitude of this vector



Cartesian Coordinates & Vectors

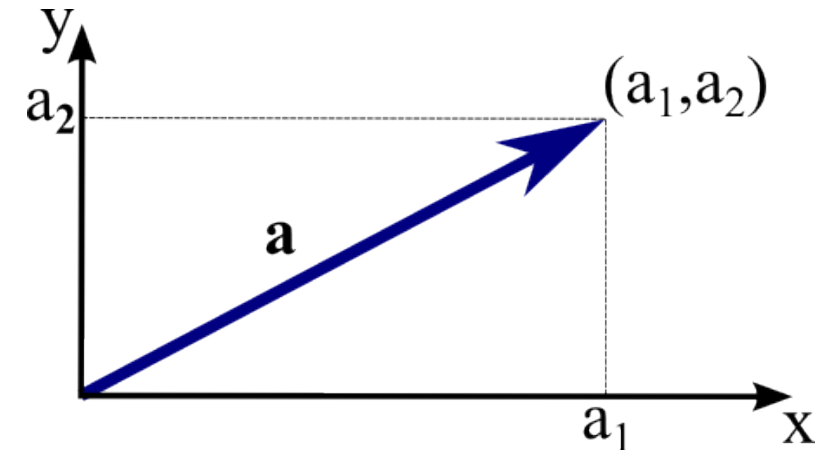
- **Cartesian Coordinates:**

- Use x, y, (and z) axes to describe positions in 2D or 3D space.
- A point is written as an ordered set:
 - 2D $\rightarrow (x, y)$
 - 3D $\rightarrow (x, y, z)$



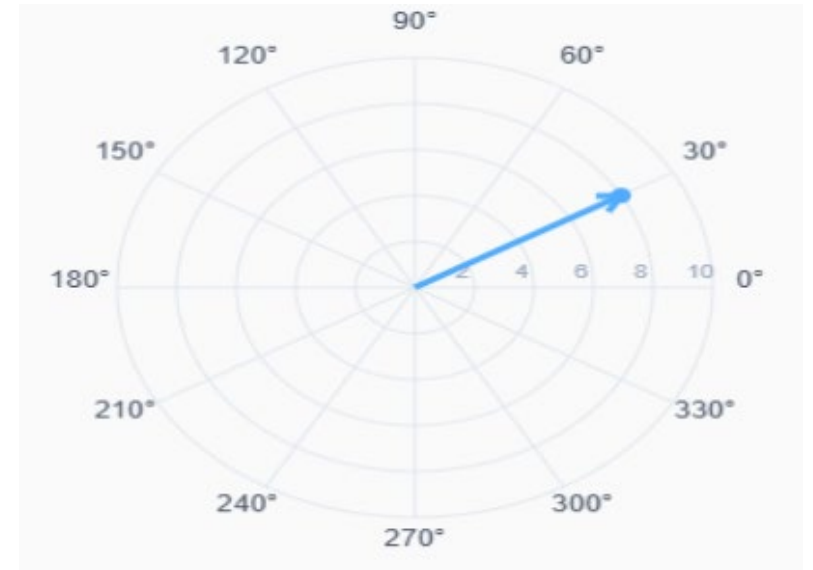
- **Vectors in Cartesian Form:**

- Expressed in terms of components along each axis.
- Example (2D): $v = (3, 4) \rightarrow$ 3 units in x, 4 units in y.
- Example (3D): $F = (2, -1, 5) \rightarrow$ 2 in x, -1 in y, 5 in z.



Polar Coordinates & Vectors

- Polar Coordinates:
 - Describe a point using radius (r) and angle (θ) instead of x and y .
 - A point is written as: (r, θ) .
 - Example: $(5, 30^\circ) \rightarrow$ 5 units from origin, at 30° counter-clockwise from x-axis.
- Vectors in Polar Form:
 - Expressed by magnitude and direction.
 - Example: displacement = 10 m at 45° .
 - It can also be written as magnitude $\angle$ angle ($10\angle 45^\circ$)



Cartesian to polar

- To find the magnitude we use the calculation we looked at earlier
- To find the angle we just put use $S_h^o C_h^a T_a^o$ we know the adjacent and opposite values as they are x and y
- So, we can use tan to find the angle
- Make sure you are using the right option on your calculator too, either radians or degrees depending on the question

$$\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan(x) = \frac{\text{opposite}}{\text{adjacent}}$$

Convert Cartesian to Polar

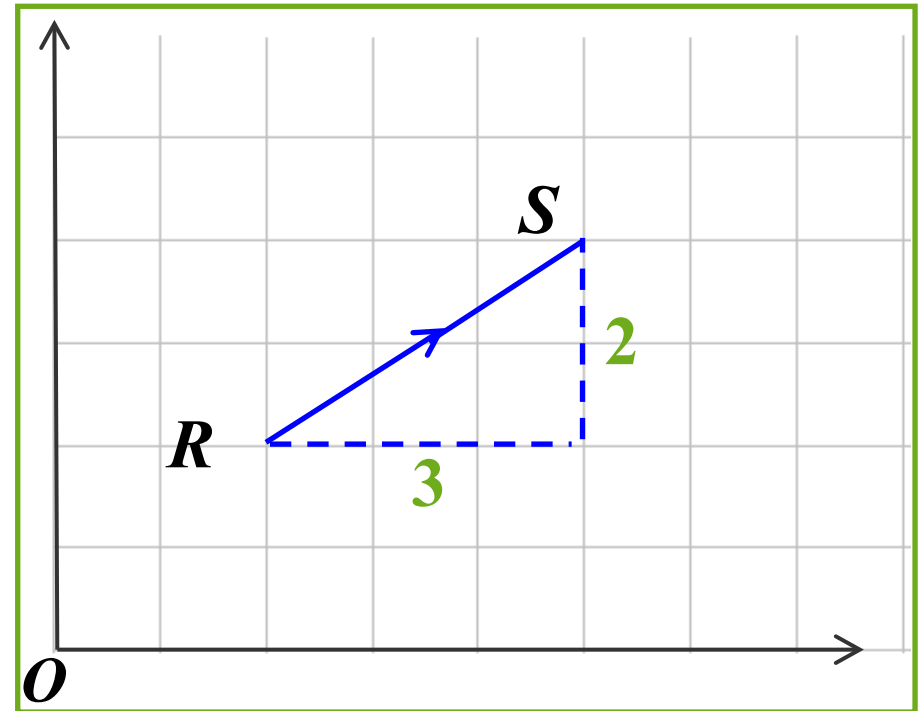
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

note: may need to add 180°

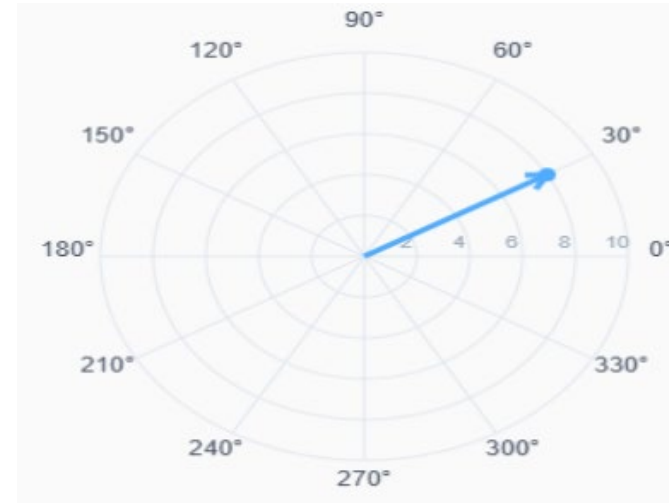
Cartesian to polar

- So, we know that $RS^2 = 3^2 + 2^2$
- $RS \approx 3.61$
- $\theta = \tan^{-1} \left(\frac{2}{3} \right) = 33.69006753^\circ$
- So our final polar form is:
- $3.61 \angle 33.69^\circ$



Polar to Cartesian

- To convert polar to cartesian again we just use $S_h^o C_h^a T_a^o$
- As we have the hypotenuse and the angle we can just cos and sin to work out x and y
- $h * \sin \theta = y$
- $h * \cos \theta = x$



Polar to Cartesian

- So, for the graph on this slide we have:
- $8 \angle 30^\circ$
- $x = 8 * \cos(30) = 6.92820323$
- $y = 8 * \sin(30) = 4$
- So, we can write: $(6.928, 4)$

